**Math without a calculator Key and Guide**

**Round 1: Scientific Notation**

1. Write the following numbers in scientific notation:
	1. Twenty-three thousand = 23,000 🡪 2.3 x 104
	2. 70 trillion = 70,000,000,000,000 🡪 7 x 1013
	3. 15 = 1.5 x 101
	4. 0.00348 = 3.48 x 10-3
2. 2.9 x 1011 + 3.7 x 1013 = 2.9 x 1011 + 370 x 1011

370 + 2.9 = 372.9

372.9 x 1011 🡪 3.73 x 1013

1. 2 x 103 • 4 x 105

2 • 4 = 8 x 103+5 = 8 x 108

1. 5.0 x 10-4 / 2.5 x 10-6

5.0/2.5 = 2 x 10-4 – (-6) = 2 x 102

1. Five hundred billion times thirty-five thousand

500,000,000,000 • 35,000 = 5 x 1011 • 3.5 x 104

 5.0

x3.5 17.5 x 1015 🡪 1.75 x 1016

 250

+1500

 17.50

**Round 2: Percentages**

1. Thirteen percent of a 12,000-acre forest is being logged. How many acres will be logged?

13% of 12,000 = 0.13 x 12,000

 12,000

 x 0.13

 36000

 +120000

 1560.00 🡪 1560 acres

1. If 35% of a natural area is to be developed, leaving 520 acres untouched, how many acres is the natural area?

If 35% is going to be developed, then 65% is going to be untouched.

 0.65 x A = 520 🡪 A = 520/0.65 = 800 acres total in the natural area

1. If the concentration of mercury in a water supply changes from 65 ppm to 39 ppm in a ten-year period, what is the percentage change of the mercury concentration?

Percent change = (New – Original)/Original x 100

Percent change = (39 ppm – 65 ppm)/65 ppm x 100 🡪 -26 ppm/65 ppm x 100 = 40%

1. In 2000, the level of ammonia in a river was 50 ppm. In 2004, the level was 84 ppm. What is the percentage increase since 2000?

Percent change = (New – Original)/Original x 100

Percent change = (84 ppm – 50 ppm)/50 ppm x 100 🡪 34 ppm/50 ppm x 100 = 68%

1. The average cost of electricity in Connecticut currently 10.1 cents/kWh. This summer prices are expected to decrease by 25%. What will be the new cost per kWh for electricity this fall?

10.1 cents/kWh x 0.25 10.100

 -2.525

 10.1 $7.575 🡪 $7.58

 0.25

 505

 +2020

 2.525

**Round 3: Metric Units**

1. Convert 14,000 mm to meters.

14,000 mm x 1 m/1000 mm = 14 m

1. Convert 4.66 megabytes to kilobytes.

4.66 megabytes = 4.66 x 106 bytes x 1 kilobyte/1000 bytes = 4.66 x 103 kilobytes

= 4660 kilobytes

1. Convert 7500 megawatts to watts.

7500 megawatts x 106 W/1 megawatt = 7500 x 106 watts = 7.5 x 109 watts

1. Convert 0.09 mm3 to cm3.

(1 cm)3

(10 mm)3

0.09 mm3 x

1 cm3

1000 mm3

0.09 mm3 x 🡪 0.00009 cm3

1. Convert 225 cm2 to km2.

100 cm = 1 m 1000 m = 1 km 🡪 100,000 cm = 1 km 🡪 105 cm = 1 km

(1 km)2

225 cm2 x

(105 cm)2

1 km2

225 cm2 x 🡪 2.25 x 102 /1010 km2 = 2.25 x 10-8 km2

1010 cm2

**Round 4: Unit Conversions**

1 barrel = 150 L 1 metric ton = 1000 kg

1 L = 0.26 gallons 1 ton = 2000 lbs

1. Fifty-eight thousand kilograms of solid waste is equivalent to how many metric tons of garbage?

1 metric ton

58,000 x = 58 tons

1000 kg

1. Your community is installing a windmill to supplement its energy needs. It is rated to produce 6 MW. How many homes can this supply energy if 1 home uses 1200 W?

6 MW = 6,000,000 W x 1 home/1200 W = 5000 Homes

1. If a tectonic plate moves 25 km in a million years, what is its rate of movement in cm/year?

100 cm = 1 m 1000 m = 1 km 🡪 100,000 cm = 1 km

=

10 yr

25 cm

=

1my

x

25 km

105 cm

x

2.5 cm/yr

106 yr

1 km

1 my

1. The BP oil spill in the Gulf of Mexico lasted for 90 days. It is estimated that the spill released 50,000 barrels of oil/day. How many gallons was spilled into the Gulf in total during this time?

50,000 barrels/day x 90 days = 4,500,000 barrels x 150 L/barrel x 0.26 gallons/L

 4.5 x 106  x 1.5 x 102 = 6.75 x 108 L • 2.6 x 10-1 gallons/L = 17.6 x 107 gallons

1. Most energy used in the US is produced from coal. It is estimated that one person is responsible for the burning of 1 ton of coal per year to meet their energy needs. For every 100 lbs of coal burned, 2.5 lbs of sulfur go into the atmosphere. How much sulfur is put into the air by coal burning each year for the entire US, which has an approximate population of 300,000,000?

300,000,000 Americans x 1 ton of coal/year/American = 300,000,000 tons of coal/year

1.5 x 1010 lbs of sulfur /year

x

2.5 lbs of sulfur

x

2000 lbs of coal

300,000,000 tons of coal/year x

100 lbs of coal

1 ton

**APES Math Prep**

This year in APES you will hear the two words most dreaded by high school students…NO CALCULATORS! That’s right, you cannot use a calculator on the AP Environmental Science exam even though one of the free response questions will require calculations of some sort. Since the regular tests you will take are meant to help prepare you for the APES exam, you will not be able to use calculators on regular tests all year either. The good news is that most calculations on the tests and exams are written to be fairly easy calculations and to come out in whole numbers or to only a few decimal places. The challenge is in setting up the problems correctly and knowing enough basic math to solve the problems. With practice, you will be a math expert by the time the exam rolls around. So bid your calculator a fond farewell, tuck it away so you won’t be tempted, and start sharpening your math skills!

*Directions*

*Read each section for review. Look over the examples and use them for help on the practice problems. Then complete each practice problem making sure to do the following…*

1. *Write out all your work, even if it’s something really simple. This is required on the APES exam so it will be required on all your assignments, labs, quizzes, and tests as well.*
2. *Include units in each step. Your answers always need units and it’s easier to keep track of them if you write them in every step.*
3. *Check your work. Go back through each step to make sure you didn’t make any mistakes in your calculations. Also check to see if your answer makes sense. For example, a person probably will not eat 13 million pounds of meat in a year. If you get an answer that seems unlikely, it probably is. Go back and check your work.*

**Section 1: Decimals**

Part I: The basics

Decimals are used to show fractional numbers. The first number behind the decimal is the tenths place, the next is the hundredths place, the next is the thousandths place. Anything beyond that should be changed into scientific notation (which is addressed in another section.)

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Part II: Adding or Subtracting Decimals

To add or subtract decimals, make sure you line up the decimals and then fill in any extra spots with zeros. Add or subtract just like usual. Be sure to put a decimal in the answer that is lined up with the ones in the problem.

* *

**Part III: Multiplying Decimals

Line up the numbers just as you would if there were no decimals. DO NOT line up the decimals. Write the decimals in the numbers but then ignore them while you are solving the multiplication problem just as you would if there were no decimals at all. After you have your answer, count up all the numbers behind the decimal point(s). Count the same number of places over in your answer and write in the decimal.

**Part IV: Dividing Decimals

*Scenario One*: If the divisor (the number after the / or before the ) does not have a decimal, set up the problems just like a regular division problem. Solve the problem just like a regular division problem. When you have your answer, put a decimal in the same place as the decimal in the dividend (the number before the / or under the ).

*Scenario Two*: If the divisor does have a decimal, make it a whole number before you start. Move the decimal to the end of the number, then move the decimal in the dividend the same number of places.



Then solve the problem just like a regular division problem. Put the decimal above the decimal in the dividend. (See Scenario One problem).

**Section 2: Averages**

To find an average, add all the quantities given and divide the total by the number of quantities.

*Example*: Find the average of 10, 20, 35, 45, and 105.

*Step 1: Add all the quantities*. 10 + 20 + 35 + 45 + 105 = 215

*Step 2: Divide the total by the number of given quantities*. 215 / 5 = 43

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

**Section 3: Percentages**

Introduction:

Percents show fractions or decimals with a denominator of 100. Always move the decimal TWO places to the right go from a decimal to a percentage or TWO places to the left to go from a percent to a decimal.

*Examples:* .85 = 85%. .008 = .8%

Part I: Finding the Percent of a Given Number

To find the percent of a given number, change the percent to a decimal and MULTIPLY.

*Example: 30% of 400*

*Step 1:* 30% = .30

*Step 2:* 400 x .30 = 12000

*Step 3:* *Count the digits behind the decimal in the problem and add decimal to the answer.* 12000 🡪 120.00 🡪 120

Part II: Finding the Percentage of a Number

To find what percentage one number is of another, divide the first number by the second, then convert the decimal answer to a percentage.

*Example:* What percentage is 12 of 25?

*Step 1:* 12/25 = .48

*Step 2:* .48 = 48% (12 is 48% of 25)

Part III: Finding Percentage Increase or Decrease

To find a percentage increase or decrease, first find the percent change, then add or subtract the change to the original number.

*Example:* Kindles have dropped in price 18% from $139. What is the new price of a Kindle?

*Step 1:* $139 x .18 = $25

*Step 2:* $139 - $25 = $114

Part IV: Finding a Total Value

To find a total value, given a percentage of the value, DIVIDE the given number by the given percentage.

*Example:* If taxes on a new car are 8% and the taxes add up to $1600, how much is the new car?

*Step 1:* 8% = .08

*Step 2:* $1600 / .08 = $160,000 / 8 = $20,000 *(Remember when the divisor has a decimal, move it to the end to make it a whole number and move the decimal in the dividend the same number of places.)*

Section 4: Metric Units

Kilo-, centi-, and milli- are the most frequently used prefixes of the metric system. You need to be able to go from one to another without a calculator. You can remember the order of the prefixes by using the following sentence: *King Henry Died By Drinking Chocolate Milk*. Since the multiples and divisions of the base units are all factors of ten, you just need to move the decimal to convert from one to another.

*Example*: 55 centimeters = ? kilometers

*Step 1: Figure out how many places to move the decimal. King Henry Died By Drinking… – that’s six places. (Count the one you are going to, but not the one you are on.)*

*Step 2: Move the decimal five places to the left since you are going from smaller to larger.*

*55 centimeters = .00055 kilometers*

*Example: 19.5 kilograms = ? milligrams*

*Step 1: Figure out how many places to move the decimal. Henry Died By Drinking Chocolate Milk – that’s six places. (Remember to count the one you are going to, but not the one you are on.)*

*Step 2: Move the decimal six places to the right since you are going from larger to smaller. In this case you need to add zeros.*

*19.5 kilograms = 19,500,000 milligrams*

Section 5: Scientific Notation

Introduction:

Scientific notation is a shorthand way to express large or tiny numbers. Since you will need to do calculations throughout the year WITHOUT A CALCULATOR, we will consider anything over 1000 to be a large number. Writing these numbers in scientific notation will help you do your calculations much quicker and easier and will help prevent mistakes in conversions from one unit to another. Like the metric system, scientific notation is based on factors of 10. A large number written in scientific notation looks like this:

1.23 x 1011

The number before the x (1.23) is called the coefficient. The coefficient must be greater than 1 and less than 10. The number after the x is the base number and is always 10. The number in superscript (11) is the exponent.

Part I: Writing Numbers in Scientific Notation

To write a large number in scientific notation, put a decimal after the first digit. Count the number of digits after the decimal you just wrote in. This will be the exponent. Drop any zeros so that the coefficient contains as few digits as possible.

*Example*: 123,000,000,000

*Step 1: Place a decimal after the first digit.* 1.23000000000

*Step 2: Count the digits after the decimal…there are 11.*

*Step 3: Drop the zeros and write in the exponent*. 1.23 x 1011

Writing tiny numbers in scientific notation is similar. The only difference is the decimal is moved to the left and the exponent is a negative. A tiny number written in scientific notation looks like this:

4.26 x 10-8

To write a tiny number in scientific notation, move the decimal after the first digit that is not a zero. Count the number of digits before the decimal you just wrote in. This will be the exponent as a negative. Drop any zeros before or after the decimal.

*Example*: .0000000426

*Step 1*: 00000004.26

*Step 2: Count the digits before the decimal…there are 8.*

*Step 3: Drop the zeros and write in the exponent as a negative*. 4.26 x 10-8

Part II: Adding and Subtracting Numbers in Scientific Notation

To add or subtract two numbers with exponents, the exponents must be the same. You can do this by moving the decimal one way or another to get the exponents the same. Once the exponents are the same, add (if it’s an addition problem) or subtract (if it’s a subtraction problem) the coefficients just as you would any regular addition problem (review the previous section about decimals if you need to). The exponent will stay the same. Make sure your answer has only one digit before the decimal – you may need to change the exponent of the answer.

*Example*: 1.35 x 106 + 3.72 x 105 = ?

*Step 1: Make sure both exponents are the same. It’s usually easier to go with the larger exponent so you don’t have to change the exponent in your answer, so let’s make both exponents 6 for this problem.*

3.72 x 105 🡪 .372 x 106

*Step 2: Add the coefficients just as you would regular decimals. Remember to line up the decimals.*

1.35

+ .372

1.722

*Step 3: Write your answer including the exponent, which is the same as what you started with.*

1.722 x 106

Part III: Multiplying and Dividing Numbers in Scientific Notation

To multiply exponents, multiply the coefficients just as you would regular decimals. Then add the exponents to each other. The exponents DO NOT have to be the same.

*Example*: 1.35 x 106  X 3.72 x 105 = ?

*Step 1: Multiply the coefficients.* 1.35

 x 3.72

 270

 9450

 40500

 50220 🡪 5.022

*Step 2: Add the exponents.* 5 + 6 = 11

*Step 3*: *Write your final answer.* 5.022 x 1011

To divide exponents, divide the coefficients just as you would regular decimals, then subtract the exponents. In some cases, you may end up with a negative exponent.

*Example*: 5.635 x 103 / 2.45 x 106 = ?

*Step 1: Divide the coefficients.* 5.635 / 3.45 = 2.3

*Step 2: Subtract the exponents.* 3 – 6 = -3

*Step 3: Write your final answer.* 2.3 x 10-3

Section 6: Dimensional Analysis

Dimensional analysis is a way to convert a quantity given in one unit to an equal quantity of another unit by lining up all the known values and multiplying. It is sometimes called factor-labeling. The best way to start a factor-labeling problem is by using what you already know. In some cases you may use more steps than a classmate to find the same answer, but it doesn’t matter. Use what you know, even if the problem goes all the way across the page!

In a dimensional analysis problem, start with your given value and unit and then work toward your desired unit by writing equal values side by side. Remember you want to cancel each of the intermediate units. To cancel a unit on the top part of the problem, you have to get the unit on the bottom. Likewise, to cancel a unit that appears on the bottom part of the problem, you have to write it in on the top.

Once you have the problem written out, multiply across the top and bottom and then divide the top by the bottom.

*Example*: 3 years = ? seconds

3 years

*Step 1: Start with the value and unit you are given. There may or may not be a number on the bottom.*

*Step 2: Start writing in all the values you know, making sure you can cancel top and bottom. Since you*

*have years on top right now, you need to put years on the bottom in the next segment. Keep*

*going, canceling units as you go, until you end up with the unit you want (in this case seconds)*

*on the top.*

x

x

x

x

3 years 365 days 24 hours 60 minutes 60 seconds

 1 year 1 day 1 hour 1 minute

*Step 3: Multiply all the values across the top. Write in scientific notation if it’s a large number. Write units on your answer.*

3 x 365 x 24 x 60 x 60 = 9.46 x 107 seconds

*Step 4: Multiply all the values across the bottom. Write in scientific notation if it’s a large number.*

*Write units on your answer if there are any. In this case everything was cancelled so there are no units.*

1 x 1 x 1 x 1 = 1

*Step 5: Divide the top number by the bottom number. Remember to include units.*

9.46 x 107 seconds / 1 = 9.46 x 107 seconds

*Step 6: Review your answer to see if it makes sense. 9.46 x 107 is a really big number. Does it make sense for there to be a lot of seconds in three years? YES! If you had gotten a tiny number, then you would need to go back and check for mistakes.*

In lots of APES problems, you will need to convert both the top and bottom unit. Don’t panic! Just convert the top one first and then the bottom.

*Example:* 50 miles per hour = ? feet per second

50 miles

1 hr

*Step 1: Start with the value and units you are given. In this case there is a unit on top and on bottom.*

5280 feet

1 mile

x

50 miles

1 hr

*Step 2: Convert miles to feet first.*

*Step 3: Continue the problem by converting hours to seconds.*

1 min

60 sec

x

1 hour

60 min

x

5280 feet

1 mile

50 miles

1 hr

x

*Step 4: Multiply across the top and bottom. Divide the top by the bottom. Be sure to include units on*

 *each step. Use scientific notation for large numbers.*

50 x 5280 feet x 1 x 1 = 264000 feet

1 x 1 x 60 x 60 seconds = 3600 seconds

264000 feet / 3600 seconds = 73.33 feet/second

Modified from *Kara House, Carmel High School, Carmel, Indiana*